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NUCLEAR STRUCTURE OF EVEN-EVEN $^{104-110}$Mo ISOTOPES

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Abstract—From the E-Gamma Over Spin (E-GOS) curve and the energy level ratios, we have deduced that the $^{104-110}$Mo isotopes lies between the vibrational U(5) and axial deformed rotor SU(3) limits and most of them display the X(5) symmetry features. We have compared the results obtained by interacting boson model IBM-2 for the above isotopes with those of the X(5) limit and then given a clear description about the validity of the Hamiltonian parameters used in this study.

Index Terms— Critical point symmetry, interacting boson model (IBM-2), even-even Mo isotopes.

Atomic nuclei are known to exhibit changes of their energy levels and electromagnetic transition rates among them when the number of protons and/or neutrons is modified, resulting in phase shape transition from one kind of collective behavior to another. These transitions are not phase transition of the usual thermodynamic type. They are quantum phase transitions [1] (initially called ground state phase transition) occurring in the Hamiltonians.

In the framework of the Interacting Boson Model [2], which describes the nuclear structure of even-even nuclei within the U(6) symmetry, possessing the U(5), SU(3), and O(6) limiting dynamical symmetries, appropriate for vibrational, axial deformed, and γ-unstable nuclei respectively.

Shape phase transitions have been studied 25 years ago [3] using the classical limit of the model [4,5], pointing out that there is a second order shape phase transition between U(5) and O(6), a first order shape phase transition between U(5) and SU(3). It is instructive to place these shape phase transitions on the symmetry triangle of the IBM [6], at the three corners of which the three limiting symmetries of the IBM appear. The X(5) critical point symmetry, introduced by Iachello [7,8] has an analytical solution to describe the transition from a spherical harmonic vibrator U(5) to an axially deformed rotor SU(3), which has stimulated considerable efforts both experimentally and theoretically. Initial work concentrated on the nuclei in the rare –earth region with N=90. Extensive studies have shown that ^{152}Sm [9] and ^{150}Nd [10] are close manifestation of the X(5) critical point symmetry. Additional examples of X(5) behavior have been suggested in ^{154}Gd [11] and ^{104}Mo [12]. Later experiments provided that the yrast band energies as well as the B(E2) values are in good agreement with the X(5) predictions for ^{162}Yb[13], ^{166}Hf [14] and ^{176}Os [15]. Lifetimes of the first 4^+ and 6^+ states in ^{104}Mo and ^{106}Mo have been measured using the recoil distance following spontaneous fission of ^{252}Cf [12]. Excitation energies and electromagnetic transition strength in even- even $^{96-108}$Mo have been described systematically by using the proton-neutron boson model IBM-2 [16]. The dynamical symmetry X(5) arises when the potential in the Bohr Hamiltonian [17] is decoupled into two components-an infinite square well potential for the quadrupole deformation parameter β and a harmonic potential well for the triaxiality deformation parameter γ [18,19]. The experimental signature for X(5) behavior are the following [19]. (a) the energies of the yrast states $E(J_1^+)$, should show characteristic ratios between those of a vibrator and a rotor; (b) the strength of transitions between yrast states as reflected in the B(E2;J→J-2) values should increase with angular momentum at a rate between the values for a vibrator and rotor; (c) the position of the first excited collective 0_2^+ is 5.67 times the energy of 2_1^+; (d) the B(E2;J→J-2) values for intrasequence transitions should be

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lower for the nonyrast sequence relative to those of the yrast sequence.

The aim of this paper is to know how the shape changes in $^{104-110}$Mo isotopes by using the E-Gamma Over Spin (E-GOS) curve [20], and the energy level ratios. Then determine the energy levels and electromagnetic transition strength for the above isotopes using the proton-neutron IBM-2.

2 THEORETICAL FRAMEWORK

IBM Hamiltonian takes different forms depending on the applicable regions U(5), SU(3), and O(6) of the traditional IBM triangle. The Hamiltonian that is considered is in the form [21]

$$H = H_{sd} + \sum\theta_L[d^+d^+d^+)^{(L)}[d^\sim d^\sim d^\sim]^{(L)} \quad (1)$$

Where H_{sd} is the standard Hamiltonian of the IBM

$$H_{sd} = \epsilon_d n_d + kQ.Q + k' L.L + k'' P^+.P + q_3T_3.q_3T_3 + q_4T_4.q_4T_4 \quad (2)$$

In the Hamiltonian, $\epsilon_d n_d$ and $P^+.P$ terms the characteristics of U(5) and O(6) structure respectively.

In the IBM-2 model, the degrees of freedom of neutrons and protons are explicitly taken into a count. Thus the Hamiltonian [22] can be written as:

$$H = \varepsilon_\nu n_{d\nu} + \varepsilon_\pi n_{d\pi} + kQ_\pi.Q_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\nu\nu} \quad (3)$$

Where $n_{d\nu(\pi)}$ is the neutron (proton) d-boson number operator

$$n_{d\rho} = d^+d^\sim, \rho = \nu, \pi \ , d^\sim_{\rho m} = (-1)^m d_{\rho-m} \quad (4)$$

Where $s^+_\rho, d^+_{\rho m}$ and $s_\rho, d_{\rho m}$ represent the s- and d-boson creation and annihilation operators. The rest of the operators in equation (3) are defined as:

$$Q_\rho = \left(s^+_\rho d^\sim_\rho + d^+_\rho s_\rho\right)^2 + \chi_\rho(d^+_\rho d^\sim_\rho)^2$$

$$V_{\rho\rho} = \sum_{L=0,2,4} C_{L\rho}\left((d^+_\rho d^+_\rho)^{(L)}(d^+_\rho d^\sim_\rho)^{(L)}\right)^{(0)}; \rho = \pi, \nu \quad (5)$$

And

$$M_{\pi\nu} = \sum_{L=1,3}\xi_L (d^+_\nu d^+_\pi)^{(L)}(d_\nu d_\pi)^{(L)} + \xi_2(s_\nu d^\sim_\pi - s_\pi d^\sim_\nu)^{(2)}.(s^+_\nu d^+_\pi - s^+_\pi d^+_\nu)^{(2)} \quad (6)$$

In this case, $M_{\pi\nu}$ affects only the position of non-fully symmetric states relative to symmetric state. For this reason $M_{\pi\nu}$ is referred to as the Majorana force [22].

The electric quadrupole (E2) transitions are one of the most important factors within the collective nuclear structure. In IBM-2 model, the general linear E2 operator is expressed as [2]:

$$T(E2) = e_\nu Q_\nu(E2) + e_\pi Q_\pi(E2) = e_\nu T_\nu + e_\pi T_\pi \quad (7)$$

Where

$T(E2)$ is an absolute transition probability of the electric quadrupole (E2) transition.

e_π and e_ν are the proton and neutron effective charges respectively .

3 RESULTS AND DISCUSSION

Energy Gamma Over Spin (E-Gos) is a new modern and active method used to determine the symmetry limit of nuclei. This method is employed in this study through plotting a curve between E_γ/J or R and angular momentum J for the three limits U(5), SU(3) and O(6) and also with the critical point symmetry X(5) as shown in Fig. 1. It is shown that the considered isotopes lie in the transitional region U(5) to SU(3) and may be analogues to the X(5) limit feature.

The energy level ratios $R_{4/2}= (E4_1^+) / (E2_1^+)$ are characteristics of different collective motions of the nucleus [18]. Fig. 2 shows $R_{4/2}$ ratios as a function of neutron number changing from 62 to 68. A harmonic vibrator should have $R_{4/2}$ =2.00, an axial symmetric rotor has $R_{4/2}$ =3.33, γ-unstable has $R_{4/2}$ =2.5, while X(5) behavior should have $R_{4/2}$=2.91. The searched nuclei in the present study have 2.5< $R_{4/2}$ < 3.33. From these ratios, we deduced that the considered isotopes have an X(5) critical point symmetry features.

The $^{104-110}$Mo isotopes have N_π=4 and N_ν varies from 6 to 9. Whereas the parameters K, K_π and $C_{L\rho}$ with L=0, 2 were treated as free parameter and their values were estimated by fitting with the measured energies. This procedure was made by traditional values of parameters and then allowing one parameter to vary keeping the others constant until the best fit was obtained. Table1 shows the most appropriate Hamiltonian parameters of calculations for examining $^{104-110}$Mo nuclei.

Fig. 3, shows that how the strength of the quadrupole parameter K decreases with increasing neutron number.

In Table 2, we present the calculated data and available experimental ones for $^{104-110}$Mo nuclei. In addition, Fig.4 shows that the energy ratio of yrast J^+ states and yrast 2^+ state as a function of angular momentum J for $^{104-110}$Mo. ^{104}Mo and ^{110}Mo not only shows a behavior very close to X(5) predictions, but there also seems to be a trend from vibrational to rotational behavior, suggesting the possibility of a phase transition; furthermore, the ^{106}Mo and ^{108}Mo nuclei are analogous the X(5) prediction.

The reduced transition strength $B(E2; J \rightarrow J-2)$ which is normalized to their respective $B(E2; 2_1^+ \rightarrow 0_1^+)$ values are calculated and compared to the experimental data, and with X(5), and are presented in Table 3.

The proton and neutron effective charges e_π and e_ν where determined using the experimental reduced transition probability $B(E2; 2_1^+ \rightarrow 0_1^+)$ for each isotope except the ^{110}Mo nucleus because it has no available experimental B(E2) data.

In Fig.5 it is seen that the relative calculated B(E2) values for ^{104}Mo and ^{106}Mo are very close to SU(3) symmetry, while the relative experimental B(E2) values are far from it.

Table.1. The most appropriate IBM-2 Hamiltonian parameters (all in MeV), except e_ν and e_π in e.b unit for $^{104-110}$Mo isotopes

Parameters	^{104}Mo	^{106}Mo	^{108}Mo	^{110}Mo
N_ν	6	7	8	9
N_π	4	4	4	4
ε (ED)	0.01	0.008	0.07	0.05
K (RKAP)	-0.15	-0.16	-0.18	-0.20
K_ν (RKNN)	-0.06	-0.07	-0.08	-0.09
K_π (RKPP)	-0.08	-0.08	-0.05	-0.08
χ_ν (CHN)	-0.23	-0.24	-0.20	-0.18
χ_π (CHP)	0.0	0.0	0.0	0.0
$C_{L\nu}$ (CLN) (L=0)	-0.1	-0.1	-0.2	-0.05
$C_{L\nu}$ (CLN) (L=2)	-0.1	-0.1	-0.2	-0.05
$\xi 1$ (RMAJ1)	0.25	0.23	0.20	0.20
$\xi 2$ (RMAJ2)	0.25	0.23	0.20	0.20
$\xi 3$ (RMAJ3)	0.25	0.23	0.20	0.20
e_ν	0.09	0.09	0.09	---
e_π	0.122	0.125	0.178	---

Table 2. The calculated and experimental energy values of $^{104-110}$Mo nuclei with $R_{4/2}$ ratios. The experiments are taken from Refs. [23, 24, 25, 26]

	^{104}Mo		^{106}Mo		^{108}Mo		^{110}Mo	
J^π	EXP	IBM	EXP	IBM	EXP	IBM	EXP	IBM
2_1^+	0.192	0.197	0.171	0.173	0.192	0.191	0.213	0.212
4_1^+	0.560	0.544	0.522	0.510	0.563	0.537	0.599	0.554
6_1^+	1.079	1.058	1.033	1.063	1.090	1.190	1.130	1.267
8_1^+	1.721	1.666	1.688	1.702	1.752	1.915	1.782	1.982
10_1^+	2.455	2.453	2.472	2.582	2.529	2.979	2.531	3.15
$R_{4/2}$	2.916	2.76	3.05	2.95	2.93	2.80	2.81	2.61

Table 3. The calculated and experimental B (E2; J→J-2) normalized to their respective $B(E2; 2_1^+ \rightarrow 0_1^+)$ values for $^{104-110}$Mo are compared with X(5) limit [27]. The experiments are taken from Refs. [18,25,26]

J^π	X(5)	^{104}Mo		^{106}Mo	
		EXP	IBM	EXP	IBM
2_1^+	1.0	1.0	1.0	1.0	1.0
4_1^+	1.596	1.195	1.45	1.37	1.47
6_1^+	1.979	1.187	1.59	1.27	1.592
8_1^+	2.27	0.88	1.6	0.875	1.63
10_1^+	2.55	0.96	1.55	0.90	1.61

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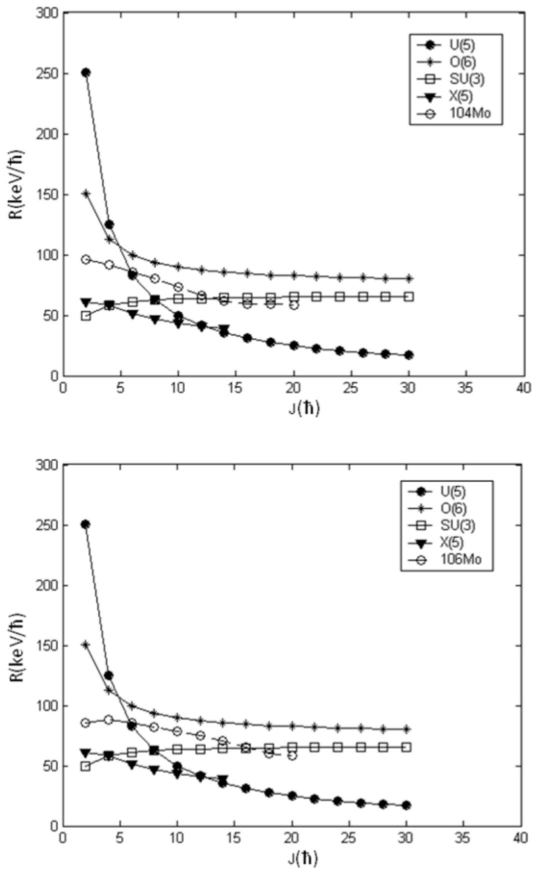


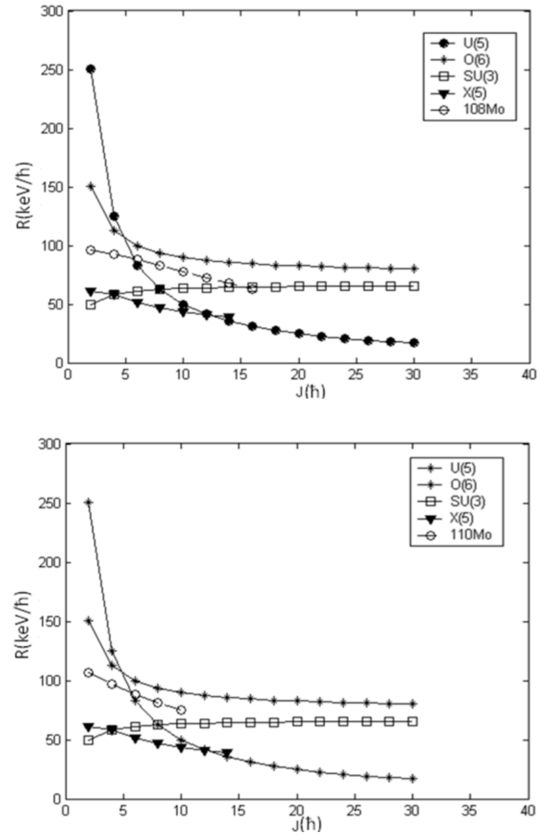


Fig.1 E-Gos energy plot for three limiting cases, U(5), SU(3) and O(6) with X(5) behavior and $^{104-110}$Mo isotopes

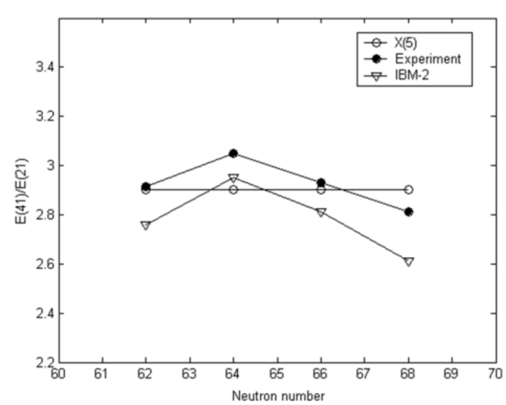


Fig.2. The $R_{4/2}$ ($E4_1^+$/ $E2_1^+$) ratios as a function of neutron number changing from 62 to 68

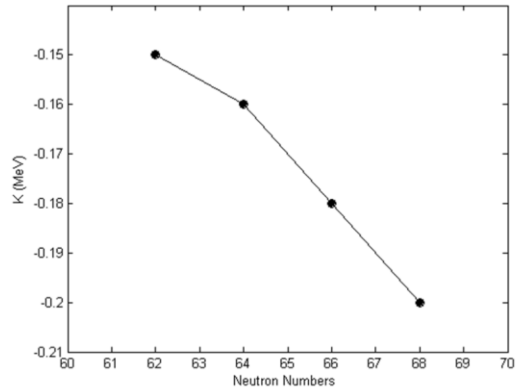
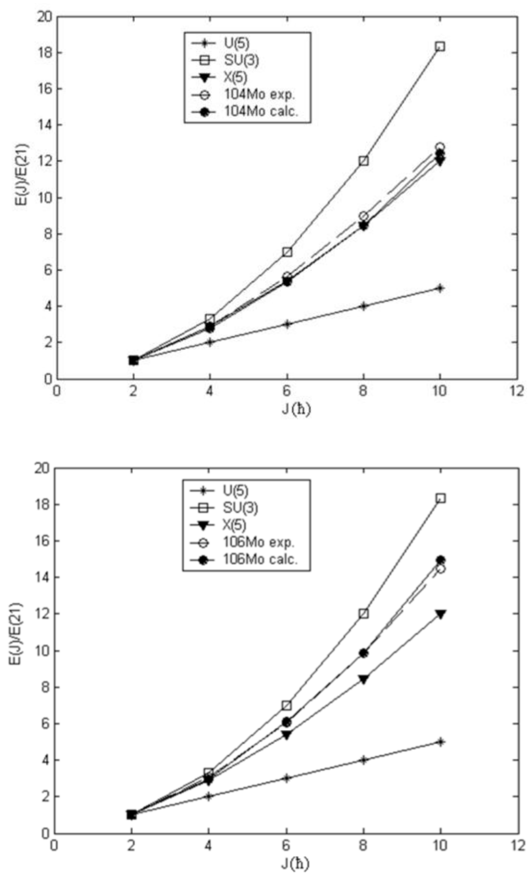


Fig.3. The quadrupole strength K versus neutron numbers



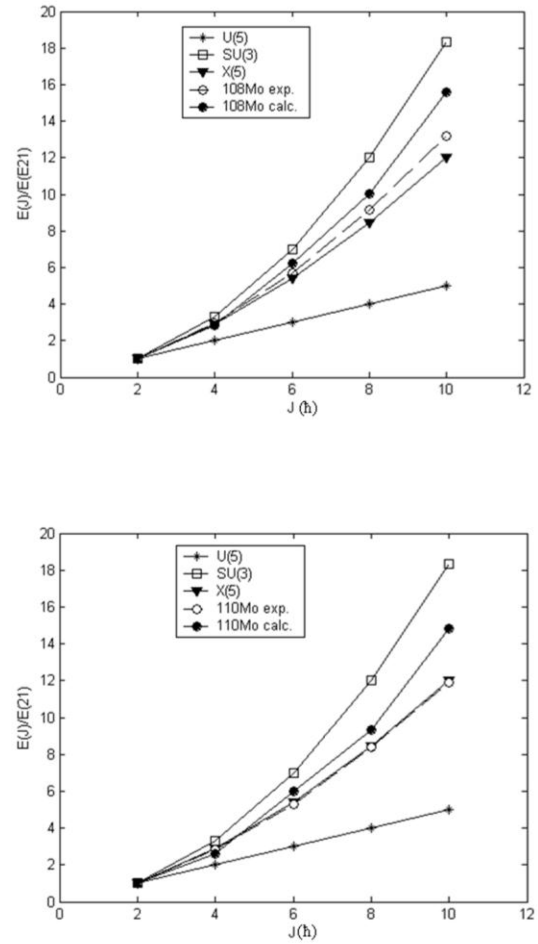


Fig.4. The energies of the yrast sequences (normalized to the energy of their respectively 2_1^+ levels) in $^{104-110}$Mo nuclei

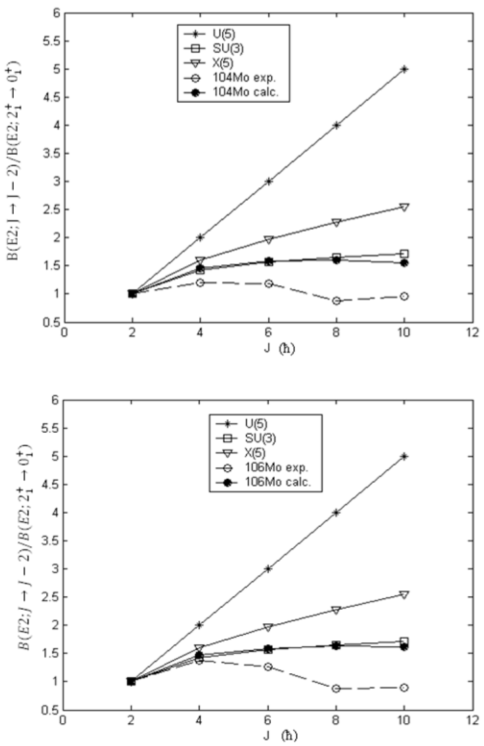


Fig.5. The reduced transition strength $B(E2; J \rightarrow J - 2)$ which is normalized to their respective $B(E2; 2_1^+ \rightarrow 0_1^+)$ values versus angular momentum J

4 CONCLUSITION

It is shown that from $E(J^+)/E(2_1^+)$ and E-Gos curve values in the yrast bands of ^{104}Mo,^{106}Mo,^{108}Mo and ^{110}Mo they follow a transitional region between U(5) and SU(3), and close to the critical point symmetry X(5) prediction. The energy levels and reduced transition probability values of the ground state band of even-even $^{104-110}$Mo are calculated by interacting boson model IBM-2 and compared them with the available experimental data and with X(5) limit.

The relative calculated B(E2) values for ^{104}Mo and ^{106}Mo are very close to the SU(3) symmetry and follow X(5) at low angular momentum J, while the relative experimental B(E2) values are far away from a rotational interpretation. Our investigations suggest that future study should focus on more detailed measurements of the excited states in Mo isotopes to get detailed information on states Beta and gamma bands.

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